Міністерство освіти і науки України  
НТУУ «КПІ ім. Ігоря Сікорського»  
Кафедра диференціальних рівнянь

Лабораторна робота №6  
Тема: “Середньокватратичне наближення функцій.

Ортогональні поліноми”  
Варіант № 6

Виконав студент 2-го курсу  
ТЕФ, групи ТР-71  
Зуєв Михайло Олександрович  
  
Перевірив: Сігайов А.О.

Київ – 2018

# Код программи:

*""" Лабораторна работа номер 6  
 з курсу Чисельні методи, варіант 6  
 Завдання: Побудувати поліном Pn(x) найкращого середньоквадратичного  
 наближення для функції f(x), яку задано таблично, з використанням  
 ортогональних поліномів.  
 Для оцінювання похибки значень функція f(x) задана.  
 f(x) = x^2 + sin(x)  
 Виконав студент 2 курсу: Зуєв Михайло Олександрович  
"""*import numpy as np  
import matplotlib.pyplot as plt  
import math  
  
  
def polynomial(xData, yData, n):  
 *"""  
 Вычисляет коэфициенты полинома n-й степени, полученного с помощью  
 ортогональных полиномов в методе наименших квадратов.* ***:param*** *xData: Таблица аргументов функции* ***:param*** *yData: Таблица значений функции* ***:param*** *n: степень полинома* ***:return****: список коэфициэнтов c полинома c0 + c1\*x \* c2\*x^2 + ... + cn\*x^n  
 """* a = np.zeros((n + 1, n + 1), dtype=np.float64)  
 b = np.zeros((n + 1), dtype=np.float64)  
 s = np.zeros((2 \* n + 1), dtype=np.float64)  
  
 for i in range(len(xData)):  
 temp = yData[i]  
 for j in range(n + 1):  
 b[j] = b[j] + temp  
 temp \*= xData[i]  
  
 temp = 1.0  
 for j in range(2 \* n + 1):  
 s[j] = s[j] + temp  
 temp \*= xData[i]  
  
 for i in range(n + 1):  
 for j in range(n + 1):  
 a[i, j] = s[i+j]  
 return np.linalg.solve(a, b)  
  
  
def my\_function(x):  
 return x \* x + math.sin(x)  
  
  
def calculate\_polynomial(Pn, x):  
 *"""  
 Считает значение полинома Pn(x) в точке х* ***:param*** *Pn: список коэфициенттов полинома* ***:param*** *x: точка в которой нужно посчитать значение* ***:return****: значение полинома Pn(x) в точке х  
 """* result = 0  
 t = len(Pn)  
 for i in range(t):  
 result += Pn[i] \* (x \*\* i)  
 return result  
  
  
# Границы функции  
a = 1  
b = 3  
  
# Шаг  
H = (b - a) / 10  
  
# Количество точек  
m = 11  
arrSize = m  
  
# Таблица значений функций  
Xt = np.zeros(arrSize)  
Yt = np.zeros(arrSize)  
  
# Создание таблицы  
for i in range(arrSize):  
 Xt[i] = a + i\*H  
 Yt[i] = my\_function(Xt[i])  
  
# Максимальный степень полинома (включитльно)  
n = 4  
  
# для графика основной функции  
h1 = (b - a) / 20  
H2 = 2\*H  
start = a - H2  
end = b + H2  
xlist = []  
ylist1 = []  
while start <= end:  
 f = my\_function(start)  
 xlist.append(start)  
 ylist1.append(f)  
 start += h1  
plt.subplot(2, 1, 1)  
plt.plot(xlist, ylist1, 'k', label='f(x)')  
  
for i in range(n + 1):  
 # Получаем коэфициенты полинома i-й степени  
 Pn = polynomial(Xt, Yt, i)  
 print(Pn)  
 # Выводим таблицу  
 print("For n = {0}".format(i))  
 print("----------------------------------------------------------------------")  
 print("| | | | | f(xj) - Pn(xj) |")  
 print("| xj | f(xj) | Pn(xj) | f(xj) - Pn(xj) | -------------- \* 100 |")  
 print("| | | | | Pn(xj) |")  
 print("----------------------------------------------------------------------")  
 start = a - H2  
 end = b + H2  
 ylist2 = []  
 while start <= end:  
 f = my\_function(start)  
 p = calculate\_polynomial(Pn, start)  
 ylist2.append(p)  
 print("|{0:5.2f} | {1:8.3f} | {2:8.3f} | {3:14.9f} | {4:21.16f}|".format(start, f, p, p - f, (p-f) \* 100 / f))  
 start += h1  
 plt.plot(xlist, ylist2, '--', label='P{0}(x)'.format(i))  
 print("----------------------------------------------------------------------\n")  
  
plt.xlabel(r'$x$')  
plt.ylabel(r'$y$')  
plt.title(r'$y = f(x), y = Pn(x)$')  
plt.legend(loc='best', ncol=2)  
  
# Additional task  
plt.subplot(2, 1, 2)  
plt.xlabel(r'$x$')  
plt.ylabel(r'$y$')  
plt.title(r'$y = f(x), y = Pn(x), y = f\'(x), y = P\'(x)$')  
plt.plot(xlist, ylist1, label='f(x)')  
Pn = polynomial(Xt, Yt, 3)  
ylist = [calculate\_polynomial(Pn, x) for x in xlist]  
plt.plot(xlist, ylist, '--', label='P3(x)')  
P1 = [Pn[1], Pn[2], Pn[3]]  
ylist = [calculate\_polynomial(P1, x) for x in xlist]  
plt.plot(xlist, ylist, ':', label='P3\'(x)')  
ylist = [2\*x + math.cos(x) for x in xlist]  
plt.plot(xlist, ylist, label='f\'(x)')  
  
plt.legend()  
plt.show()

# Результати роботи:

For n = 0

----------------------------------------------------------------------

| | | | | f(xj) - Pn(xj) |

| xj | f(xj) | Pn(xj) | f(xj) - Pn(xj) | -------------- \* 100 |

| | | | | Pn(xj) |

----------------------------------------------------------------------

| 0.60 | 0.925 | 5.138 | 4.213289124 | 455.6668383343396727|

| 0.70 | 1.134 | 5.138 | 4.003713911 | 352.9934293586717899|

| 0.80 | 1.357 | 5.138 | 3.780575507 | 278.5249598287589947|

| 0.90 | 1.593 | 5.138 | 3.544604688 | 222.4656262795354564|

| 1.00 | 1.841 | 5.138 | 3.296460613 | 179.0123569797449079|

| 1.10 | 2.101 | 5.138 | 3.036724238 | 144.5228250894956545|

| 1.20 | 2.372 | 5.138 | 2.765892512 | 116.6040023626162423|

| 1.30 | 2.654 | 5.138 | 2.484373412 | 93.6242297623801534|

| 1.40 | 2.945 | 5.138 | 2.192481868 | 74.4362344907770535|

| 1.50 | 3.247 | 5.138 | 1.890436611 | 58.2121487178469295|

| 1.60 | 3.560 | 5.138 | 1.578357995 | 44.3412096724305655|

| 1.70 | 3.882 | 5.138 | 1.256266787 | 32.3641233517976801|

| 1.80 | 4.214 | 5.138 | 0.924083967 | 21.9296958001267193|

| 1.90 | 4.556 | 5.138 | 0.581631510 | 12.7654346493763509|

| 2.00 | 4.909 | 5.138 | 0.228634171 | 4.6571668226013694|

| 2.10 | 5.273 | 5.138 | -0.135277769 | -2.5653783006194777|

| 2.20 | 5.648 | 5.138 | -0.510564806 | -9.0389507137458782|

| 2.30 | 6.036 | 5.138 | -0.897773614 | -14.8743781017482473|

| 2.40 | 6.435 | 5.138 | -1.297531583 | -20.1622097172716437|

| 2.50 | 6.848 | 5.138 | -1.710540546 | -24.9769658151560243|

| 2.60 | 7.276 | 5.138 | -2.137569774 | -29.3803775818089434|

| 2.70 | 7.717 | 5.138 | -2.579448282 | -33.4238863766422085|

| 2.80 | 8.175 | 5.138 | -3.037056552 | -37.1505927172182737|

| 2.90 | 8.649 | 5.138 | -3.511317731 | -40.5967916723525946|

| 3.00 | 9.141 | 5.138 | -4.003188410 | -43.7931939053372759|

| 3.10 | 9.652 | 5.138 | -4.513649065 | -46.7659052178994301|

| 3.20 | 10.182 | 5.138 | -5.043694259 | -49.5372186113674431|

| 3.30 | 10.732 | 5.138 | -5.594322708 | -52.1262592985074917|

----------------------------------------------------------------------

For n = 1

----------------------------------------------------------------------

| | | | | f(xj) - Pn(xj) |

| xj | f(xj) | Pn(xj) | f(xj) - Pn(xj) | -------------- \* 100 |

| | | | | Pn(xj) |

----------------------------------------------------------------------

| 0.60 | 0.925 | 0.054 | -0.870405145 | -94.1342377823036145|

| 0.70 | 1.134 | 0.417 | -0.716859339 | -63.2029765749048309|

| 0.80 | 1.357 | 0.780 | -0.576876724 | -42.5000283633421887|

| 0.90 | 1.593 | 1.144 | -0.449726523 | -28.2256277994896401|

| 1.00 | 1.841 | 1.507 | -0.334749579 | -18.1783792343951696|

| 1.10 | 2.101 | 1.870 | -0.231364935 | -11.0110472459633115|

| 1.20 | 2.372 | 2.233 | -0.139075642 | -5.8631260618480834|

| 1.30 | 2.654 | 2.596 | -0.057473722 | -2.1659115070902799|

| 1.40 | 2.945 | 2.959 | 0.013755753 | 0.4670170539541181|

| 1.50 | 3.247 | 3.322 | 0.074831515 | 2.3042842393606424|

| 1.60 | 3.560 | 3.685 | 0.125873918 | 3.5362077586086769|

| 1.70 | 3.882 | 4.049 | 0.166903730 | 4.2997975836292621|

| 1.80 | 4.214 | 4.412 | 0.197841929 | 4.6950422948423576|

| 1.90 | 4.556 | 4.775 | 0.218510491 | 4.7957879574801545|

| 2.00 | 4.909 | 5.138 | 0.228634171 | 4.6571668226014049|

| 2.10 | 5.273 | 5.501 | 0.227843250 | 4.3207700383381402|

| 2.20 | 5.648 | 5.864 | 0.215677232 | 3.8183122904653177|

| 2.30 | 6.036 | 6.227 | 0.191589443 | 3.1742677372832575|

| 2.40 | 6.435 | 6.590 | 0.154952494 | 2.4077908575835392|

| 2.50 | 6.848 | 6.954 | 0.105064550 | 1.5341312274061110|

| 2.60 | 7.276 | 7.317 | 0.041156341 | 0.5656839185825506|

| 2.70 | 7.717 | 7.680 | -0.037601148 | -0.4872268636247915|

| 2.80 | 8.175 | 8.043 | -0.132088399 | -1.6157625702831502|

| 2.90 | 8.649 | 8.406 | -0.243228559 | -2.8121348945346840|

| 3.00 | 9.141 | 8.769 | -0.371978218 | -4.0692849214187508|

| 3.10 | 9.652 | 9.132 | -0.519317853 | -5.3806508121543262|

| 3.20 | 10.182 | 9.495 | -0.686242028 | -6.7400043754686836|

| 3.30 | 10.732 | 9.859 | -0.873749458 | -8.1413413566160102|

----------------------------------------------------------------------

For n = 2

----------------------------------------------------------------------

| | | | | f(xj) - Pn(xj) |

| xj | f(xj) | Pn(xj) | f(xj) - Pn(xj) | -------------- \* 100 |

| | | | | Pn(xj) |

----------------------------------------------------------------------

| 0.60 | 0.925 | 0.962 | 0.037585125 | 4.0648278547152286|

| 0.70 | 1.134 | 1.168 | 0.033978768 | 2.9957889532076072|

| 0.80 | 1.357 | 1.386 | 0.028450123 | 2.0959955022016117|

| 0.90 | 1.593 | 1.615 | 0.021729963 | 1.3638106995115451|

| 1.00 | 1.841 | 1.856 | 0.014477448 | 0.7861892848411171|

| 1.10 | 2.101 | 2.108 | 0.007273533 | 0.3461596991642301|

| 1.20 | 2.372 | 2.373 | 0.000615169 | 0.0259341773654518|

| 1.30 | 2.654 | 2.648 | -0.005089668 | -0.1918054072413086|

| 1.40 | 2.945 | 2.936 | -0.009526049 | -0.3234157801677494|

| 1.50 | 3.247 | 3.235 | -0.012475242 | -0.3841496775223041|

| 1.60 | 3.560 | 3.546 | -0.013816893 | -0.3881614565955935|

| 1.70 | 3.882 | 3.868 | -0.013530234 | -0.3485678109457839|

| 1.80 | 4.214 | 4.202 | -0.011694287 | -0.2775204164908990|

| 1.90 | 4.556 | 4.548 | -0.008487076 | -0.1862712346405976|

| 2.00 | 4.909 | 4.905 | -0.004183847 | -0.0852229236717319|

| 2.10 | 5.273 | 5.274 | 0.000845683 | 0.0160373498370240|

| 2.20 | 5.648 | 5.655 | 0.006141016 | 0.1087194886754716|

| 2.30 | 6.036 | 6.047 | 0.011155479 | 0.1848247900290827|

| 2.40 | 6.435 | 6.451 | 0.015261683 | 0.2371497280924498|

| 2.50 | 6.848 | 6.866 | 0.017757793 | 0.2592956899044803|

| 2.60 | 7.276 | 7.293 | 0.017874539 | 0.2456812055247679|

| 2.70 | 7.717 | 7.732 | 0.014782906 | 0.1915534324641756|

| 2.80 | 8.175 | 8.183 | 0.007602412 | 0.0929960004266389|

| 2.90 | 8.649 | 8.645 | -0.004590090 | -0.0530692329356654|

| 3.00 | 9.141 | 9.118 | -0.022751191 | -0.2488884451901519|

| 3.10 | 9.652 | 9.604 | -0.047861367 | -0.4958914913593304|

| 3.20 | 10.182 | 10.101 | -0.080915182 | -0.7947176917391726|

| 3.30 | 10.732 | 10.609 | -0.122911351 | -1.1452519428279591|

----------------------------------------------------------------------

For n = 3

----------------------------------------------------------------------

| | | | | f(xj) - Pn(xj) |

| xj | f(xj) | Pn(xj) | f(xj) - Pn(xj) | -------------- \* 100 |

| | | | | Pn(xj) |

----------------------------------------------------------------------

| 0.60 | 0.925 | 0.849 | -0.076115964 | -8.2319346086941447|

| 0.70 | 1.134 | 1.085 | -0.048759078 | -4.2989170494775637|

| 0.80 | 1.357 | 1.329 | -0.028400422 | -2.0923339034526842|

| 0.90 | 1.593 | 1.579 | -0.013918764 | -0.8735661425668497|

| 1.00 | 1.841 | 1.837 | -0.004264490 | -0.2315806303054347|

| 1.10 | 2.101 | 2.103 | 0.001533815 | 0.0729968251210833|

| 1.20 | 2.372 | 2.376 | 0.004363556 | 0.1839580294211843|

| 1.30 | 2.654 | 2.659 | 0.005023169 | 0.1892993773392510|

| 1.40 | 2.945 | 2.950 | 0.004218038 | 0.1432052418014747|

| 1.50 | 3.247 | 3.250 | 0.002557354 | 0.0787485227562779|

| 1.60 | 3.560 | 3.560 | 0.000551926 | 0.0155054004505261|

| 1.70 | 3.882 | 3.880 | -0.001387020 | -0.0357326131333590|

| 1.80 | 4.214 | 4.211 | -0.002948050 | -0.0699609993044416|

| 1.90 | 4.556 | 4.552 | -0.003918729 | -0.0860068266459698|

| 2.00 | 4.909 | 4.905 | -0.004183847 | -0.0852229236719490|

| 2.10 | 5.273 | 5.269 | -0.003722664 | -0.0705957988160745|

| 2.20 | 5.648 | 5.646 | -0.002605221 | -0.0461223835417319|

| 2.30 | 6.036 | 6.035 | -0.000987734 | -0.0163648532906878|

| 2.40 | 6.435 | 6.436 | 0.000892865 | 0.0138741302911890|

| 2.50 | 6.848 | 6.851 | 0.002725197 | 0.0397927769798997|

| 2.60 | 7.276 | 7.280 | 0.004130452 | 0.0567720578350101|

| 2.70 | 7.717 | 7.722 | 0.004670069 | 0.0605136577842859|

| 2.80 | 8.175 | 8.179 | 0.003854024 | 0.0471440986617366|

| 2.90 | 8.649 | 8.650 | 0.001149628 | 0.0132916522221372|

| 3.00 | 9.141 | 9.137 | -0.004009254 | -0.0438595465099860|

| 3.10 | 9.652 | 9.639 | -0.012212640 | -0.1265351284849618|

| 3.20 | 10.182 | 10.158 | -0.024064638 | -0.2363535847145517|

| 3.30 | 10.732 | 10.692 | -0.040173505 | -0.3743249469091052|

----------------------------------------------------------------------

For n = 4

----------------------------------------------------------------------

| | | | | f(xj) - Pn(xj) |

| xj | f(xj) | Pn(xj) | f(xj) - Pn(xj) | -------------- \* 100 |

| | | | | Pn(xj) |

----------------------------------------------------------------------

| 0.60 | 0.925 | 0.920 | -0.004188747 | -0.4530126241072432|

| 0.70 | 1.134 | 1.132 | -0.002605060 | -0.2296789932221358|

| 0.80 | 1.357 | 1.356 | -0.001427715 | -0.1051835586516216|

| 0.90 | 1.593 | 1.593 | -0.000616120 | -0.0386687524582809|

| 1.00 | 1.841 | 1.841 | -0.000114843 | -0.0062364816313993|

| 1.10 | 2.101 | 2.101 | 0.000139793 | 0.0066529647318563|

| 1.20 | 2.372 | 2.372 | 0.000213909 | 0.0090179477613273|

| 1.30 | 2.654 | 2.654 | 0.000171108 | 0.0064482393609804|

| 1.40 | 2.945 | 2.946 | 0.000068391 | 0.0023219304473947|

| 1.50 | 3.247 | 3.247 | -0.000046981 | -0.0014466993929910|

| 1.60 | 3.560 | 3.559 | -0.000139682 | -0.0039241135672976|

| 1.70 | 3.882 | 3.881 | -0.000187513 | -0.0048307339755510|

| 1.80 | 4.214 | 4.214 | -0.000181619 | -0.0043100396165957|

| 1.90 | 4.556 | 4.556 | -0.000125692 | -0.0027586484700775|

| 2.00 | 4.909 | 4.909 | -0.000034200 | -0.0006966309408005|

| 2.10 | 5.273 | 5.273 | 0.000070373 | 0.0013345295931187|

| 2.20 | 5.648 | 5.649 | 0.000161210 | 0.0028540379960164|

| 2.30 | 6.036 | 6.036 | 0.000211773 | 0.0035086714529013|

| 2.40 | 6.435 | 6.436 | 0.000201257 | 0.0031273070546435|

| 2.50 | 6.848 | 6.849 | 0.000120861 | 0.0017647940327989|

| 2.60 | 7.276 | 7.275 | -0.000019195 | -0.0002638342923159|

| 2.70 | 7.717 | 7.717 | -0.000181992 | -0.0023582160181439|

| 2.80 | 8.175 | 8.175 | -0.000295623 | -0.0036161842628303|

| 2.90 | 8.649 | 8.649 | -0.000244394 | -0.0028256086437410|

| 3.00 | 9.141 | 9.141 | 0.000140393 | 0.0015358438854658|

| 3.10 | 9.652 | 9.653 | 0.001090005 | 0.0112935365841592|

| 3.20 | 10.182 | 10.185 | 0.002908069 | 0.0285619270278943|

| 3.30 | 10.732 | 10.738 | 0.005980513 | 0.0557246629509838|

